

Newton's Second Law

(K. M. Westerberg, 6/2003)

Introduction

The goal of this lab is to verify Newton's Second Law, which states that the net force acting on an object is proportional to the acceleration of that object, with the mass of that object being the proportionality constant:

$$\vec{F}_{\text{tot}} = m\vec{a} \quad (1)$$

You will test this proportionality by connecting a glider cart to a hanging weight using a string, and allowing the hanging weight to exert a force (via tension in the string) on a glider cart which is free to slide across a nearly frictionless air track, and measuring the glider's acceleration. The acceleration is measured for different values of hanging weight, and the resulting tension force vs. acceleration plot is constructed. Naturally, there may be other forces acting on the glider cart aside from the tension force, such as friction and gravity (if the air track is not perfectly level). If, however, these forces are constant as we expect them to be, the tension versus acceleration plot should yield a straight line with a slope equal to the mass of the glider cart.

The acceleration can be determined by measuring the time required for the glider cart released from rest to pass between two photogates placed a certain distance apart. The equations of kinematics can then be used to solve for the acceleration, assuming that the acceleration is constant throughout its motion. The glider cart will be released some finite "lead distance" away from the first photogate; thus, the clock will not start right away when the glider cart is released. This adds some complication to the method, but reduces potential errors dramatically, and is well worth the additional effort.¹

Air track setup

The basic setup is shown in Fig. 1. The glider cart (mass M) is connected to a hanging weight (mass m) using a string which passes over a pulley. The tension in the string will exert a force on the glider cart, causing it to accelerate to the right. You will measure the acceleration with the help of two photogates (not shown in Fig. 1). Our goal in this section is to use Newton's Second Law to derive a relationship between the string tension and the glider cart acceleration that we can test experimentally.

First of all, since the glider cart will be accelerating to the right, the hanging weight must be accelerating downwards as well. In fact, both accelerations must have the same magnitude (you should understand why from your 2A lecture). This means that the tension

¹This method is a generalization of the "D and D method" used in the 4AL lab course. In that method, the lead distance and distance between photogates are equal to each other.

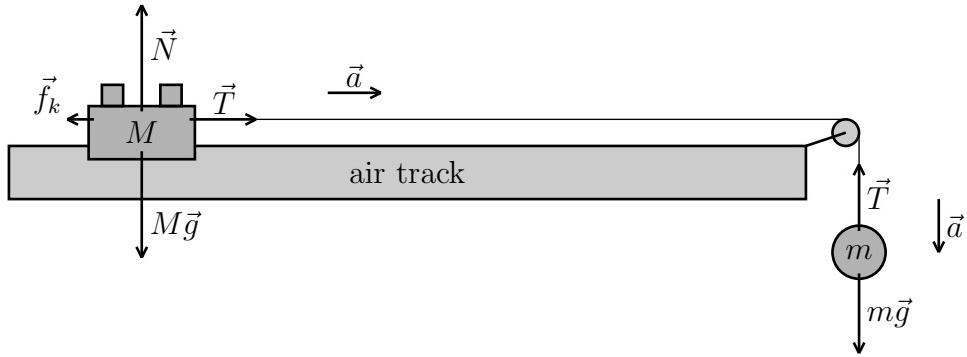


Figure 1: Air track setup.

force will not be *equal* to the weight of the hanging weight, although it should be related to it. In fact, applying Newton's Second Law to the hanging weight (vertical component) yields

$$+T - mg = -ma ,$$

which leads to the following after solving for T ,

$$T = mg - ma . \quad (2)$$

Since the acceleration will be measured during the experiment, all of the variables on the right-hand side of Eq. 2 will be known, and so the tension can be calculated.

We now apply Newton's Second Law to the glider cart. Since there is no vertical component of the glider's acceleration, Newton's Second Law tells us that the normal (upward) force from the air track is equal to the weight of the glider cart ($N = Mg$). However, this relation does not interest us nearly as much as the relationship between T and a that we can derive from the horizontal components:

$$-f_k + T = +Ma .$$

Rearranging this equation (solving for T) yields

$$T = Ma + f_k . \quad (3)$$

If Newton's Second Law is valid, then it must be the case that we get a straight line when we plot T vs. a , where the slope of the line is equal to M and the y -intercept is equal to f_k (which represents the kinetic friction force plus any other forces which might be acting on the glider cart in the horizontal direction). You will verify this experimentally by measuring the acceleration for various hanging weights, calculating the tension force using Eq. 2 and plotting that force versus acceleration on a graph. You should be able to fit a straight line to the data points which passes close to the origin (f_k should not be very large). Furthermore, the slope you obtain from the graph should be equal to the mass of the glider cart, which you can measure using an electronic balance.

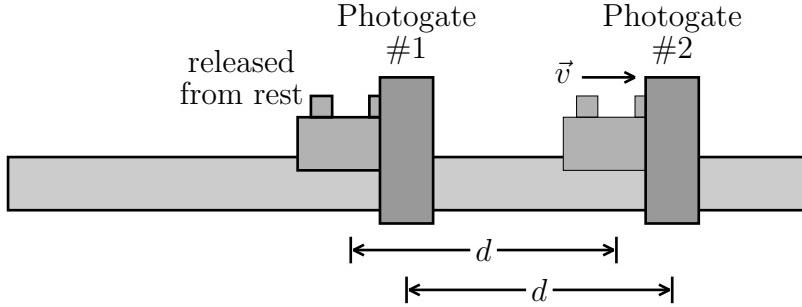


Figure 2: One way to measure the acceleration of the glider cart.

Note that we are assuming f_k is constant, independent of the hanging weight used. This is not unreasonable in light of the fact that the kinetic friction force should be proportional to the normal force between the air track and the glider cart and independent of most other factors, such as the speed of the glider cart (we will be studying friction in the next lab). Since the weight of the glider cart does not change during the experiment, the kinetic friction force should not change either, and so f_k should be constant. The value of f_k could also conceivably include contributions from the gravitational force if the air track is not perfectly level, but those forces are also expected to be constant throughout the experiment (provided, of course, that you do not change the inclination of the air track during the experiment).

Measuring the acceleration

One of the most effective ways to measure the acceleration of the glider cart is to measure the amount of time it takes for the glider cart to travel a certain distance. This time can be measured by placing two photogates a distance d apart as shown in Fig. 2. The glider flags trigger the photogate timer by blocking a light beam which passes across each of the two photogates. When the photogate is set to “pendulum mode” (as it should be for this experiment), the timer is started when the front edge of the front flag passes through the first photogate and is stopped when the same front edge of the front flag passes through the second photogate.

Suppose we start the glider from rest at a position on the air track where it is just about to trigger the first photogate, as shown in Fig. 2. It follows that the timer should start just after we release the glider. The glider then travels a distance d before reaching the second photogate, causing the timer to stop. As a result, the photogates record the amount of time, Δt , required for the glider to travel a distance d starting from rest. If acceleration is constant, then we may use the constant acceleration equations, one of which tells us that

$$d = v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a(\Delta t)^2 .$$

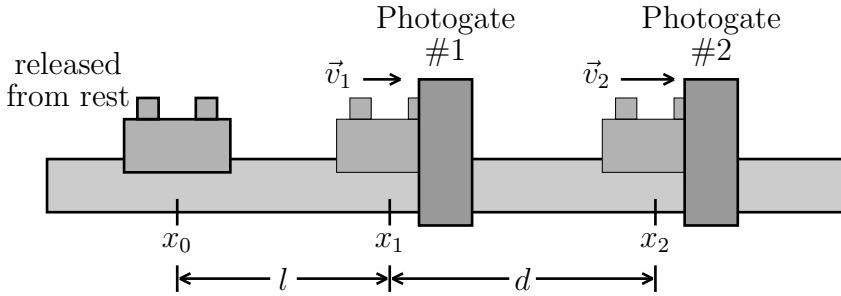


Figure 3: A more reliable way to measure the acceleration.

Solving for the acceleration yields

$$a = \frac{2d}{(\Delta t)^2} . \quad (4)$$

It turns out that there is a serious flaw with this method, which would become apparent if we tried to use it. The problem is that it is impossible to start the glider cart *exactly* at the position where it triggers the first photogate (otherwise the timer would start before we released the glider cart). Thus, there is always going to be a tiny bit of distance which the glider cart must travel before reaching the first photogate. This wouldn't be such a serious problem, except for the fact that the glider cart is started *from rest*. Since the glider cart is not moving initially, it takes an unexpectedly large amount of time to move that little bit of distance required to trigger the first photogate (try it) — time that the photogate timer should be recording but is not.

It turns out that the best solution to this problem is to give the glider cart a running start by releasing the glider cart from rest a distance l (called the *lead distance*) from where it triggers the first photogate. The constant acceleration equations can be used to determine a relationship between the acceleration and the photogate time, although it is somewhat less obvious how to work it out. Nevertheless, the problem can be solved (see the Appendix if you are curious how to do it). The result is

$$a = \frac{2(\sqrt{l+d} - \sqrt{l})^2}{(\Delta t)^2} , \quad (5)$$

where Δt is the time measured by the photogate timer. As expected, substituting $l = 0$ into Eq. 5 reduces that equation to Eq. 4.

Although Eq. 5 looks complicated, it is actually very easy to use because the acceleration numerator ($2(\sqrt{l+d} - \sqrt{l})^2$) only has to be computed once at the beginning of the lab (the values of l and d are fixed throughout the experiment). Thus it is a simple matter to calculate the acceleration of the glider cart for a given run: simply measure the time required by the glider cart to pass between the two photogates (Δt) and divide the previously calculated numerator by $(\Delta t)^2$.

Since Eq. 5 applies in the general case where the lead distance l is not zero, we can use this equation to better understand why it is a mistake to try to start the glider cart from

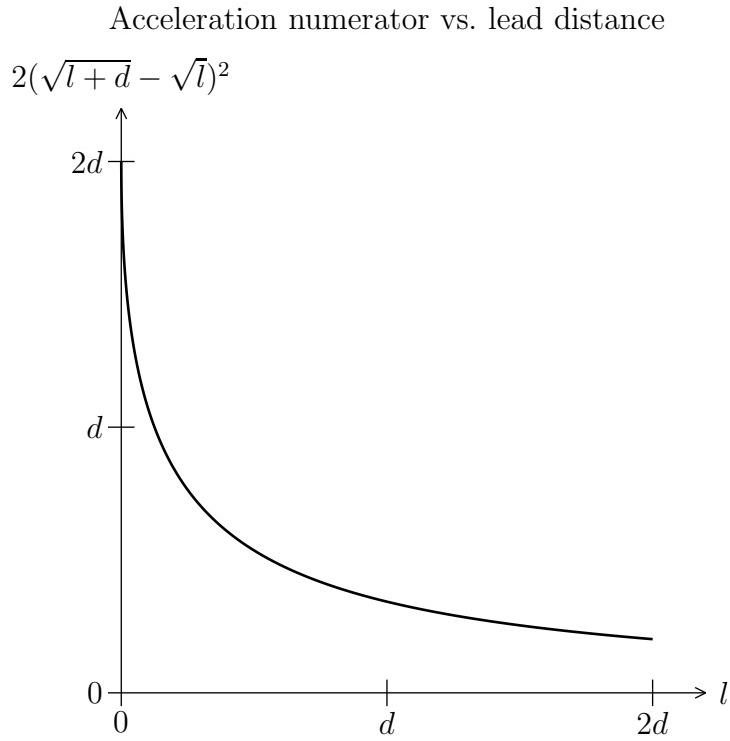


Figure 4: Acceleration numerator as a function of lead distance. Notice the “square-root” cusp at $l = 0$, which would give rise to huge errors if we tried to start the glider cart from the first photogate position.

the first photogate position. Figure 4 shows a graph of the acceleration numerator as a function of the lead distance l (d is fixed). Because of the square roots in the expression for the acceleration numerator, there is a sharp decrease in its value as l is initially increased from zero. Thus, if l is zero, or very close to zero, a slight error in the value of l can result in a huge error in the value of the acceleration. Such an error is easily avoided simply by releasing the glider cart away from the first photogate by some appreciable distance.

It turns out that the lead distance does not have to be all that large in order to eliminate the problem. In fact, making the lead distance too large will necessarily reduce the distance *between* the photogates (d) to the point where errors in the measurement of d and Δt will cause more serious problems. It turns out that the total distance ($l + d$) traveled by the glider from its release to when the photogate timer is stopped is limited by the fact that the hanging weight can only fall a certain distance before hitting the floor (the hanging weight must be suspended freely the entire time that the glider cart is accelerating). The value of $l + d$ must be less than the maximum height through which the hanging weight can fall (typically 80–100 cm). A detailed uncertainty analysis (which is well beyond the scope of this course) reveals that for a fixed value of $l + d$, the lowest uncertainty in the acceleration is obtained when the lead distance l is about 1/5 of this total distance $l + d$. Since $l + d$ should be about 80–100 cm, l itself should be in the range 15–20 cm.

Procedure

In order to test Newton's Second Law, you will measure the acceleration of the glider cart for various hanging weights, and plot tension versus acceleration. The resulting graph should be a straight line whose slope is equal to the glider cart's mass. *Best results are obtained by making sure that the air track is level before taking any data.* This insures that the force f_k in Eq. 3 is entirely due to the friction between the glider cart and the air track, and provides an additional check of Newton's Second Law (f_k should be positive and is not expected to be very large).

After verifying that the air track is level, the following procedure should be followed.

(Q-1) Determine the position of the glider cart at its release point (x_0), when it triggers the first photogate (x_1), and when it triggers the second photogate (x_2). See Fig. 3.

The values you get for x_0 , x_1 , and x_2 will be used to calculate l and d and therefore play a vital role in calculating the acceleration. Since these values only need to be measured once, it pays to do it carefully. The following procedure should work well.

- Move the glider cart towards the end of the air track with the hanging weight until the hanging weight just barely touches the floor (attach one of the 50 g weight hangers before you do this step). Place the second photogate so that the front edge of the front flag on the glider cart has clearly moved beyond it. It is important during the experimental run that the glider cart reach the second photogate *before* the hanging weight hits the floor. Give a few centimeters leeway.
- Now back up the glider cart and turn the photogate on. Move the glider cart *slowly* towards the second photogate until the front edge of the front flag triggers the photogate timer (the timer will start). You may want to do this a few times to make sure you get it right. Record the *glider position* as x_2 (use the arrow on the glider cart and read the length scale on the air track).
- Now move the glider cart to the opposite end of the air track until the hanging weight is hanging freely just below the pulley. This will be your initial glider position. Record this position as x_0 . You might want to move the glider forward just a little so that the glider position is a nice round number, since you will be releasing the glider from this position repeatedly.
- Calculate a proposed value of x_1 such that $l = \frac{1}{5}(l + d)$ (the recommended value of l). The following should work

$$x_1 \text{ (proposed)} = x_0 + \frac{1}{5}(x_2 - x_0) \quad (6)$$

Do not record this value as x_1 — you will measure x_1 more accurately once you move the first photogate into position.

- Move the glider cart to the proposed x_1 position and move the first photogate up to the glider cart so that the glider's front flag will trigger the photogate at *approximately* the position it is now. You do not need to do *this* step accurately.
- With the first photogate firmly in place, now back up the glider cart a bit and determine the glider position when it triggers the first photogate using the same procedure you used with the second photogate. Record the *glider position* as x_1 . *This step must be done accurately.*

Once you have done this, the photogates will not be moved again for the duration of the experiment. Also be careful not to move the flag to a new position on the glider cart, for this will change the position of the glider cart when the first and second photogates will be triggered. You should note that x_1 and x_2 represent the position of the *glider cart* when it triggers the photogates, and not the photogate positions themselves. There is an important difference.

(Q-2) Calculate l and d from x_0 , x_1 , and x_2 (see Fig. 3), and use these values to calculate the acceleration numerator. You will use this value repeatedly when calculating the acceleration of the glider cart.

It may be a good idea to get your instructor to check your work so far. Wrong values at this stage could spoil the results of your experiment.

(Q-3) Perform the experiment and fill in the data table provided.

You should start off with a hanging mass of 50 g (0.050 kg). Move the glider cart to its initial position x_0 , make sure the photogate timer is reset, and release the glider cart *from rest*. The glider cart should pass unimpeded through the first and second photogate (make sure the weight does not hit the floor until after the glider triggers the second photogate and stops the timer). The photogate timer should record the time between photogates. Record this time in the data table. You will repeat each time measurement a total of five times and take the average time — that is your value of Δt . Use this value of Δt to calculate the acceleration of the glider cart. Repeat this for the other hanging masses listed in the table.

(Q-4) Continue the analysis by filling in the second data table.

You should know how to calculate the hanging weight from the hanging mass. The acceleration can be copied from the previous data table. The tension is then calculated using Eq. 2.

(Q-5) Plot T versus a and determine the experimental value for the glider cart mass (M) and the friction force (f_k).

Show how you do this. Be sure to calculate the uncertainty for M (which rule do you apply?).

(Q-6) Now measure the glider cart's mass using the electronic balance.

The uncertainty can be obtained from Table 1 in "Measurement and Calculation".

(Q-7) Compare the experimental value of M obtained from the graph with the measured mass obtained from the electronic balance using the discrepancy test.

Don't forget to clean up the lab station when you are finished.

Appendix

In this Appendix, we explain how Eq. 5 is derived. The reader should refer to Fig. 3 as this is explained.

The glider cart is released from rest a distance l away from the first photogate. Define t_1 to be the time required by the glider cart to reach this first photogate. The glider must then travel an additional distance d to reach the second photogate. The total distance covered by the glider cart between its point of release and the second photogate is $l + d$. Define t_2 to be the total time required by the glider cart to reach the second photogate.

It is clear that if the glider cart's acceleration is constant throughout its motion up to the second photogate, then

$$l = \frac{1}{2}at_1^2, \quad l + d = \frac{1}{2}at_2^2.$$

Solving for t_1 and t_2 yields

$$t_1 = \sqrt{\frac{2l}{a}}, \quad t_2 = \sqrt{\frac{2(l+d)}{a}}.$$

The times t_1 and t_2 are not measured individually, but the difference in these two times is measured by the photogate timer. Thus,

$$\Delta t = t_2 - t_1 = \sqrt{\frac{2(l+d)}{a}} - \sqrt{\frac{2l}{a}} = \sqrt{\frac{2}{a}}(\sqrt{l+d} - \sqrt{l})$$

is the time required by the glider cart to move from one photogate to the other, and is therefore the time recorded on the photogate timer.

It is now a simple matter of algebra (left to the reader) to solve for a . The result is Eq. 5.

Newton's Second Law (addendum)

In this version of the lab, the air track + glider cart will be replaced by the dynamics track + cart. Students should read the previous version of the lab and answer the pre-lab quiz questions prior to arriving to the lab. This document explains the differences between the previous version and this version of the lab.

Setup

The following equipment should be set up at each lab station:

One dynamics track	One LabQuest Mini Box
One dynamics cart with flag	One computer
Two photogates	One mass set + hanger + string

Some things to note:

- Photos are available of the setup as a whole, as well as a closeup of the dynamics cart.
- The LabQuest Mini Box should be connected to both photogates and the computer.
- The computer will run LoggerPro in order to read the photogate timer (configuration described below).
- The flag should be attached to the dynamics cart with the solid bar on top and in line with the photogates.
- The flag itself should be placed on the side of the cart nearer to the photogate posts (otherwise the edge of the plastic will trigger the photogate and mess up the timer readings).
- Be careful: the flag can move back and forth along the length of the dynamics cart (this could affect the lead distance, l). To counter this, the flag should be pushed as far to the front of the cart as possible.
- The dynamics cart position can be measured by reading the location of the front of the cart on the length scale on the track. There is parallax, but students should be able to read the position within ± 1 mm or so.
- Students should position the photogates and determine x_0 (position of cart at release), x_1 (position of cart when triggering photogate 1), and x_2 (position of cart when triggering photogate 2) in the manner described in the previous version of the lab. A red LED light will light up on the photogate when that photogate is blocked.
- When I performed the lab myself, I included all four added masses with the dynamics cart, making the mass around 770 g. One could make a different choice here, but smaller mass carts accelerate more.

LoggerPro setup

I have included a file `newt2.cmb1`. If you double-click this file, LoggerPro will run with a properly configured table. To get the Δt value for a particular run:

- Carefully line up the dynamics cart with the initial position.
- Press the green “collect” button in LoggerPro.
- Release the dynamics cart from rest.
- Catch the cart after the second photogate, preferably before the hanging mass hits the (styrofoam block on the) floor.
- Read Δt from the last column in the LoggerPro table (labelled “Delta t”).

If `newt2.cmb1` is unavailable, then LoggerPro can be configured as follows:

- Run LoggerPro. A default configuration will appear.
- Right click each of the three graphs and select “delete” to delete the graph. We will not be using any graphs in LoggerPro.
- Select “Data” under the top menu and select “Delete column”. A list of calculated columns will appear (e.g., “distance 1”, “velocity 1”, etc). Select one of them to delete it. Repeat until *all* calculated columns are gone. Columns labelled “Time”, “State 1” and “State 2” cannot be deleted and should remain after this step is completed.
- Select “Data” under the top menu and select “New calculated column”.
- Under “Column Definition”, change the following:

Name: Time between photogates

Short name: Delta t

Units: s

Expression: BlockedToBlocked("Time", "GateState 1", "GateState 2")

Pay careful attention to capitalization and spacing when entering the expression.

- Under “Options”, change the color to Black (or something equally visible). Click “Done” to complete the calculated column.
- You may need to resize the table in order to be able to read all of the columns.